Kleine AG, October 2026, Münster University

The isomorphism between the Drinfeld and Lubin–Tate spaces at infinite level

organized by Carlo Kaul, Webpage: `carlo.info/kleine-ag`

Organization

The workshop will take place on October 10, 2026, at Münster University. The precise location is:

Room: to be announced
Orléans-Ring 12, Seminarraumzentrum
48149 Münster

The Kleine AG will have the following syllabus:

	Morning Session
10:45 – 11:00	**Welcome & Coffee**
11:00 – 12:00	**Talk I:** Rapoport–Zink spaces, period maps and the Drinfeld and Lubin–Tate towers
12:00 – 12:15	**Time for questions & Coffee Break**
12:15 – 13:15	**Talk II:** Versions of the Fargues–Fontaine curve and their torsors
	Lunch & Next Kleine AG
13:15 – 14:00	**Lunch Break** (in front of the seminar room)
14:00 – 14:15	**Discussion of the topic for the next Kleine AG:** Bring ideas!
	Afternoon Session
14:15 – 15:15	**Talk III:** Local shtukas, their duality and the Grothendieck–Messing period map
15:15 – 15:30	**Coffee Break**
15:30 – 16:30	**Talk IV:** Local Shimura varieties as generic fibers of Rapoport–Zink spaces
16:30 – 16:45	**Coffee Break**
16:45 – 17:45	**Talk V:** Application to the Jacquet–Langlands correspondence: A survey of [HM26]
from 18:00	**Optional: Dinner** (in the city center)

From Bonn, we recommend the following train connections:

- 07:33 Bonn Hbf – 08:02 Köln Hbf (RB26), 08:21 Köln Hbf – 10:22 Münster(Westf)Hbf (RE7), 10:39 Münster Hauptbahnhof B1 – 10:51 Münster Försterstraße (Bus 1)
- 17:56 Münster Försterstraße – 18:06 Eisenbahnstraße (B) (Bus 5), 18:35 Münster(Westf)Hbf – 20:38 Köln Hbf (RE7), 20:56 Köln Hbf – 21:25 Bonn Hbf (RB26)

  or, if you want to stay for dinner,
- 20:35 Münster(Westf)Hbf – 22:38 Köln Hbf (RE7), 22:56 Köln Hbf – 23:25 Bonn Hbf (RB26).

For speakers, funding is available both for travel and accomodation (if you join from farther away).

Introduction

In Section 5.54 of their book [RZ96] on period spaces for p-divisible group, Rapoport and Zink conjecture a certain duality on the data (G, b, μ) giving rise to moduli spaces of p-divisible groups, producing an equivariant isomorphism of the corresponding moduli spaces *at infinite level*.

Let $F/\mathbb{Q}_p$ be a finite extension, let $\breve{F}$ be the completion of its maximal unramified extension, let D be the central division algebra over F of invariant $1/n$ and $\mathcal{O}_D$ its maximal order. In the case of the so-called *Lubin–Tate* and *Drinfeld tower*, the conjecture of Rapoport and Zink takes the following form:

1. Let LT_n over $\breve{F}$ be h-dimensional Lubin–Tate space of level n, i.e. the rigid generic fiber of the moduli space of deformations of one-dimensional formal groups over $\mathcal{O}_F$ of height h, together with a trivialization of their p^n-torsion.

2. Let $\mathcal{M}_n$ over $\breve{F}$ be h-dimensional Drinfeld space of level n, i.e. the rigid generic fiber of the moduli space of deformations of special formal $\mathcal{O}_D$-modules of height $h^2[F:\mathbb{Q}_p]$, together with a trivialization of their ϖ^n-torsion, for ϖ a uniformizer of $\mathcal{O}_D$.

Both of these spaces can be viewed as generic fibers of EL-type moduli of p-divisible groups, and hence constitute examples of generic fibers of Rapoport–Zink spaces $\mathcal{M}_{\mathcal{D},n}$ for (dual) EL-data $\mathcal{D}$.

For simplicity, let us take $F = \mathbb{Q}_p$ from now on. In terms of so-called *local shtuka data*, they are then defined by the following data of (G, b, μ):

1. In the Lubin–Tate case, $G = \mathrm{GL}_n$, μ is the cocharacter of $G_{\overline{\mathbb{Q}}_p} = \mathrm{GL}_{n,\overline{\mathbb{Q}}_p}$ corresponding to $(1, 0, \ldots, 0)$ and $b \in \mathrm{B}(G, \mu)$ is (a representative of) the unique basic element.

2. In the Drinfeld case, $G = D^\times$, μ is the cocharacter of $G_{\overline{\mathbb{Q}}_p} = \mathrm{GL}_{n,\overline{\mathbb{Q}}_p}$ corresponding to $(1, \ldots, 1, 0)$ and $b \in \mathrm{B}(G, \mu)$ is (a representative of) the unique basic element.

In [SW13, Theorem 6.5.4], the authors prove that the generic fiber of any EL-type Rapoport–Zink space becomes (pre)perfectoid at infinite level, i.e. there is a perfectoid space

$$\mathcal{M}_{\mathcal{D},\infty} \sim \varprojlim_n \mathcal{M}_{\mathcal{D},n},$$

classifying moduli of p-divisible groups with EL-datum $\mathcal{D}$ together with a trivialization of the (rational) Tate module as level structure. Moreover, in [SW13, Theorem 7.2.3], they prove the existence of a natural $G(\mathbb{Q}_p) \times \check{G}(\mathbb{Q}_p)$-equivariant isomorphism $\mathcal{M}_{\mathcal{D},\infty} \cong \mathcal{M}_{\check{\mathcal{D}},\infty}$ of perfectoid spaces, where $G(\mathbb{Q}_p)$ (resp. $\check{G}(\mathbb{Q}_p)$) denotes the group of quasi-isogenies of the framing object in the defining moduli problems for the EL-data $\mathcal{D}$ (resp. $\check{\mathcal{D}}$).

One crucial aspect of the theory is the existence of period maps $\pi_{\mathrm{GM}} : \mathcal{M}_{\mathcal{D},\infty} \to \mathscr{F}\!\ell_{\mathrm{GM}}$ (Grothendieck–Messing period map) and $\pi_{\mathrm{HT}} : \mathcal{M}_{\mathcal{D},\infty} \to \mathscr{F}\!\ell_{\mathrm{HT}}$ (Hodge–Tate period map), which are interchanged under the duality isomorphism $\mathcal{M}_{\mathcal{D},\infty} \cong \mathcal{M}_{\check{\mathcal{D}},\infty}$.

Already in the paper [SW13], and in more detail in the lecture notes [SW20], the authors give an interpretation of the duality isomorphism purely in terms of p-adic Hodge theory and p-adic shtukas on the Fargues–Fontaine curve: Fix a reductive group $G/\mathbb{Q}_p$ and a triple $(\mathcal{G}, b, \mu)$ consisting of a smooth group scheme $\mathcal{G}$ over $\mathbb{Z}_p$ with reductive generic fiber G, an element $b \in G(\breve{\mathbb{Q}}_p)$ and a conjugacy class of cocharacters $\mu : \mathbb{G}_m \to G_{\overline{\mathbb{Q}}_p}$. Denote by E the reflex field of μ and by $K \subseteq G(\mathbb{Q}_p)$ a compact-open subgroup.

Let S be a perfectoid space over $\overline{\mathbb{F}}_p$. In [SW20, Prop. 23.3.1], the authors provide an interpretation of the S-points of the moduli spaces of p-adic G-shtukas with K-structure $\mathrm{Sht}_{G,b,\mu,K}$ as isomorphism classes of quadruples $(S^\sharp, \mathcal{E}, \alpha, \mathbb{P})$, for

- $S^\sharp$ an untilt of S to a perfectoid space over E,

- $\mathcal{E}$ a G-torsor on FF_S, the Fargues–Fontaine curve over S, trivial at each geometric point of S,
- $\alpha : \mathcal{E}|_{\mathrm{FF}_S\setminus S^\sharp} \xrightarrow{\sim} \mathcal{E}^b|_{\mathrm{FF}_S\setminus S^\sharp}$ is an isomorphism of G-torsors, meromorphic along $S^\sharp$ and with meromorphy bounded by μ,
- $\mathbb{P}$ a pro-étale $\underline{K}$-torsor on S such that $\mathbb{P}_\eta = \mathbb{P} \times^{\underline{K}} \underline{G(\mathbb{Q}_p)}$ is the pro-étale $\underline{G(\mathbb{Q}_p)}$-torsor corresponding to $\mathcal{E}$.

We view $\mathrm{Sht}_{G,b,\mu,K}$ as a (locally spatial) diamond over $\mathrm{Spd}(\breve{E})$. Using this moduli problem, one can easily deduce (cf. [SW20, Corollary 23.3.2]) that there is a duality of the form $\mathrm{Sht}_{G,b,\mu,K} \cong \mathrm{Sht}_{\check{G},\check{b},\check{\mu},K}$ of diamonds over $\mathrm{Spd}(\breve{E})$, for a certain duality $(G, b, \mu) \mapsto (\check{G}, \check{b}, \check{\mu})$ on (basic) local shtuka data. To recover the duality of Rapoport–Zink spaces, or finally the duality of the Lubin–Tate and Drinfeld towers, the bulk of the work then consists in verifying the following two statements:

1. For minuscule cocharacters μ, there is an étale Grothendieck–Messing period map[1] $\pi_{\mathrm{GM}} : \mathrm{Sht}_{G,b,\mu,K} \to \mathscr{F}\!\ell_{\mathrm{GM}}^{\diamondsuit}$, so $\mathrm{Sht}_{G,b,\mu,K}$ is itself the *diamond* of a smooth rigid space $\mathcal{M}_{G,b,\mu,K}\ /\breve{E}$.

This smooth rigid space is non-empty if and only if $b \in \mathrm{B}(G, \mu)$ is admissible. In this case, (G, b, μ) is called a local Shimura datum, and the system of rigid spaces $(\mathcal{M}_{G,b,\mu,K})_{K\subseteq G(\mathbb{Q}_p)}$ over $\breve{E}$, together with the étale period maps π_{GM}, is called the *local Shimura variety* for the local Shimura datum (G, b, μ). The second statement is the following:

2. For any EL-datum $\mathcal{D}$ on the moduli of p-divisible groups, there exists a local Shimura datum (G, b, μ) and a natural isomorphism $\mathcal{M}_{\mathcal{D},\breve{E}} \cong \mathcal{M}_{G,b,\mu,\mathcal{G}(\mathbb{Z}_p)}$ compatible with level structures on both sides.

In particular, specializing to the data (G, b, μ) defining the Rapoport–Zink and Lubin–Tate towers as above, and realizing that these two data are interchanged by the duality on basic local shtuka data, one obtains the central Theorem of this Kleine AG:

Theorem 1. There is a natural $\mathrm{GL}_h(\mathbb{Q}_p) \times D^\times$-equivariant isomorphism of perfectoid spaces

$$\mathrm{LT}_\infty \cong \mathcal{M}_\infty$$

between the Lubin–Tate and Drinfeld space at infinite level, interchanging the natural Grothendieck–Messing and Hodge–Tate period maps of both sides. Moreover, this isomorphism is compatible with natural Weil descent data to $\mathbb{Q}_p$ (defined in [SW20, Section 3.48]) of both sides.

Finally, we want to explain how one can use this Theorem to obtain a construction and first properties of a Jacquet–Langlands functor for $G = \mathrm{GL}_h(\mathbb{Q}_p)$: Roughly speaking, the idea is to consider the following correspondence diagram

$$\begin{tikzcd} & \mathrm{LT}_{\infty,\mathbb{C}_p} \arrow[dl, "\pi_{\mathrm{GM}}"'] \arrow[dr, "\pi_{\mathrm{HT}}"] & \\ \mathscr{F}\!\ell_{\mathrm{GM},\mathbb{C}_p} & & \mathscr{F}\!\ell_{\mathrm{HT},\mathbb{C}_p}, \end{tikzcd}$$

and to realize that π_{GM} is a $D^\times$-equivariant G-torsor, while π_{HT} is a G-equivariant $D^\times$-torsor. Moreover, everything is equivariant for the Weil group action coming from the Weil descent datum to F, so modding out by the $G \times D^\times$-action gives a diagram (cf. [HM26])

$$\begin{tikzcd} {[\mathscr{F}\!\ell_{\mathrm{HT},\mathbb{C}_p}/\underline{G}]} \arrow[r, phantom, "\cong"] \arrow[d, "f"'] & {[\mathscr{F}\!\ell_{\mathrm{GM},\mathbb{C}_p}/\underline{D^\times}]} \arrow[d, "g"] \\ {[*/\underline{G}].} & {[*/\underline{D^\times}]} \end{tikzcd}$$

[1]Note that this is already crucial for proving that $\mathrm{Sht}_{G,b,\mu,K}$, a priori a v-stack, is indeed a locally spatial diamond.

of small v-stacks with $\mathrm{W}_{\mathbb{Q}_p}$-action, for $* = \mathrm{Spa}(\mathbb{C}_p, \mathcal{O}_{\mathbb{C}_p})$. Now, one can apply ones desired six-functor-formalism D of ℓ- or p-adic sheaves in order to obtain a functor

$$\mathcal{J}_{\mathrm{D}} := g_! f^* : \quad \mathrm{D}([*/\underline{G}]) \to \mathrm{D}([*/\underline{D^\times}])$$

This functor should be defined (i.e. g should be !-able) and one expects g to be D-proper (since $\mathscr{F}\!\ell_{\mathrm{GM}}$ is proper) and D-smooth and f to be D-smooth. In this case, $\mathcal{J}$ will automatically preserve D-*suave* objects (cf. [HM24, Lemma 4.5.16]), which should be identified with certain *admissible* representations after identification of $\mathrm{D}([*/\underline{G}])$ (resp. $\mathrm{D}([*/\underline{D^\times}])$) with a certain category of G-representations (resp. $D^\times$-representations).

In the final part of the Kleine AG, we will see one precise realization of this very rough picture for smooth mod-p representations due to Hansen–Mann (cf. [HM26]). To handle smooth mod-p representation theory, one would really like to put $\mathrm{D} = \mathrm{D}_{\text{ét}}(-, \mathbb{F}_p)$, since one can identify $\mathrm{D}([*/\underline{G}]) = \mathrm{D}(\mathrm{Rep}^{\mathrm{sm}}_{\mathbb{F}_p}(G))$ (cf. [HM26, Prop. 3.9]) in this case. Unfortunately, this six-functor-formalism is not well-behaved, as g does not turn out to be a !-able map. Since $g_! = g_*$ is anyways expected, one may however still define

$$\mathcal{J} := g_* f^* : \quad \mathrm{D}(\mathrm{Rep}^{\mathrm{sm}}_{\mathbb{F}_p}(G)) \to \mathrm{D}(\mathrm{Rep}^{\mathrm{sm}}_{\mathbb{F}_p}(D^\times)),$$

which one can identify with a more classically constructed Jacquet–Langlands functor of Scholze (cf. [Sch18]).

Luckily, one can pass to a better six-functor-formalism and then compare it to the aforementioned one: In Lucas Mann's thesis [Man22], he develops a full six-functor-formalism for *solid almost* $\mathcal{O}^+/p$*-sheaves*. To any small v-stack over $*$, it attaches a symmetric monoidal stable ∞-category $\mathrm{D}^a_{\square}(\mathcal{O}^+_X/p)^\varphi$ with objects given by complexes of solid quasi-coherent almost $\mathcal{O}^+_X/p$-sheaves on the étale site of X, together with a suitable φ-module structure (for φ the Frobenius on $\mathcal{O}^+_X/p$). Importantly, almost any reasonable (*bdcs*) map is !-able for this six-functor-formalism, and there is a fully faithful embedding

$$\mathrm{RH} := - \otimes \mathcal{O}^{+,a}_X/p : \quad \mathrm{D}_{\text{ét}}(X, \mathbb{F}_p)^\dagger \hookrightarrow \mathrm{D}^a_{\square}(\mathcal{O}^+_X/p)^\varphi$$

of symmetric monoidal categories for any small v-stack X, which comes with a right adjoint $M \mapsto M^\varphi$, which is roughly the functor of φ-invariants. Here, $\mathrm{D}_{\text{ét}}(X, \mathbb{F}_p)^\dagger$ denotes the full subcategory of overconvergent sheaves, which is equal to the category of all étale sheaves for X a classifying stack as before. This embedding, also called *p-torsion Riemann–Hilbert correspondence*, is moreover compatible with the Jacquet–Langlands functor in the sense that the diagram

$$\begin{tikzcd}
\mathrm{D}(\mathrm{Rep}^{\mathrm{sm}}_{\mathbb{F}_p}(G)) \arrow[r, "\mathcal{J}"] \arrow[d, hook, "\mathrm{RH}"'] & \mathrm{D}(\mathrm{Rep}^{\mathrm{sm}}_{\mathbb{F}_p}(D^\times)) \\
\mathrm{D}^a_{\square}(\mathcal{O}^+_{[*/\underline{G}]}/p)^\varphi \arrow[r, "\mathcal{J}_{\square}"'] & \mathrm{D}^a_{\square}(\mathcal{O}^+_{[*/\underline{D^\times}]}/p)^\varphi \arrow[u, "(-)^\varphi"']
\end{tikzcd}$$

commutes for $\mathcal{J}_{\square} := \mathcal{J}_{\mathrm{D}^a_{\square}(\mathcal{O}^+_X/p)^\varphi}$. Putting everything together, we get the following second Theorem of this Kleine AG:

Theorem 2. $\mathcal{J} : \mathrm{D}(\mathrm{Rep}^{\mathrm{sm}}_{\mathbb{F}_p}(G)) \to \mathrm{D}(\mathrm{Rep}^{\mathrm{sm}}_{\mathbb{F}_p}(D^\times))$ preserves admissible representations.

Program

(I) Rapoport–Zink spaces, period maps and the two towers. [RZ96]

Start by recalling the definition of a p-divisible group X, the notion of a quasi-isogeny of p-divisible groups, the universal vector extension EX, and the ratioanlized covariant Dieduonné module $M(X)$. Noting that this defines an isocrystal, also recall the definitions of the Kottwitz set $\mathrm{B}(G)$ and its subset of μ-admissible elements $\mathrm{B}(G,\mu)$ for μ a (conjugacy class of) minuscule cocharacters of μ over $\overline{\mathbb{Q}}_p$ (as in [SW20, Def. 24.1.1]). Also give the Hodge–Tate exact sequence as in [SW20, Theorem 12.1.1] for p-divisible groups over $\mathcal{O}_C$.

Give the definition of *rational EL data* for p-divisible groups as in the Introduction. Give Theorem I of [RZ96] on representability of the corresponding EL Rapoport–Zink moduli problems; for a somewhat more modern formulation, cf. [SW20, Def. 24.3.3]. Explain how to find the Drinfeld and Lubin–Tate towers as special cases of this construction, cf. [RZ17, Introduction].

Now, introduce the Grothendieck–Messing period map: For this, explain how Grothendieck–Messing theory yields an isomorphism $\mathrm{Lie}(EX)[1/p] \cong M(\mathbb{X}) \otimes_{\breve{\mathbb{Q}}_p} R$ for X/R^+ a deformation of a p-divisible group $\mathbb{X}$ over $\mathrm{Spec}(\overline{\mathbb{F}}_p)$ to an affinoid perfectoid (R, R^+), and how then the natural surjection $\mathrm{Lie}(EX) \to \mathrm{Lie}(X)$ can be used to define varying *Hodge filtrations* Fil_X, i.e. quotients of the fixed vector space $M(\mathbb{X}) \otimes_{\breve{\mathbb{Q}}_p} R \cong \breve{\mathbb{Q}}_p^n$ for varying deformations X of $\mathbb{X}$. Also state the equivariance properties of π_{GM} as in [SW13, Prop. 6.5.5].

Finally, quickly introduce the Rapoport–Zink spaces at infinite level ([SW13, Def. 6.3.3]), state its perfectoidness ([SW13, Theorem 6.3.4]) and define the Hodge–Tate period map as in [SW13, Section 7.1], also stating its equivariance properties.

(II) Versions of the Fargues–Fontaine curve and their torsors. [SW20]

Start by carefully explaining Figure 12.1 depicting $\mathrm{Spa}(A_{\mathrm{inf}})$ (with emphasis on the radius map κ), define the $\mathcal{Y}$-curve and its versions with *restricted radius*, give Theorem 13.2.1 on the unique extension of φ-modules (cf. Def. 12.3.3) over the point x_L and also explain the equivalence stated directly before (Def. 13.4.3). Now, define the Fargues–Fontaine $\mathrm{FF}_{C^\flat}$ curve over $C^\flat$ for an algebraically closed complete extension $C^\flat/\mathbb{F}_p$ and explain that φ-modules on $\mathcal{Y}_{(0,\infty)}$ equate to vector bundles on $\mathrm{FF}_{C^\flat}$.

Then, introduce the relative version of $\mathrm{Spa}(A_{\mathrm{inf}})$, namely $S \dot{\times} \mathrm{Spa}(\mathbb{Z}_p)$, and state (but not prove) Prop. 11.2.1, 11.3.1. Give the definition of a relative Fargues–Fontaine curve as in Def. 15.2.6] and define the category of G-torsors on a relative Fargues–Fontaine curve as in Theorem 19.5.2, directly giving its classification for $S = *$ as in Theorem 22.4.3. Finally, discuss Theorem 22.5.2, Theorem 22.6.1 and Theorem 22.6.2 on G-torsors on relative Fargues–Fontaine and $\mathcal{Y}$-curves and their extensions, which will become crucial in the next talk.

(III) Local shtukas, their duality and the Grothendieck–Messing period map. [SW20]

Motivate the concept of a (mixed-characteristic) shtuka as in Section 1.1, 1.2, 11.1; in particular, give Def. 11.1.1. Give the definition of a mixed-characteristic shtuka as in Def. 11.4.1.

Moving towards moduli spaces of local shtukas, give Def. 23.1.1. Immediately move to the case of one leg and give a proof sketch of Prop. 23.3.1, explain the definition of the tower of local shtuka spaces directly following it and carefully explain Corollary 23.3.2 on the duality of local shtuka data at infinite level.

Quickly introduce the most important aspects of the B_{dR}^+-affine and Beilinson–Drinfeld Grassmannians: In particular, Definition 19.1.1, Proposition 19.1.2 (omitting the part from *Moreover* to *Finally*), Prop. 19.2.1 and Def. 19.2.2 on Schubert varieties inside the Grassmannian, Prop. 19.2.3 and Theorem 19.2.4, Corollary 19.3.4 on their geometry will be relevant. Since we will mainly be interested in shtukas with one leg (and also not their integral models) for this Kleine AG, we only need the Beilinson–Drinfeld Grassmannian over $\mathrm{Spd}(\mathbb{Q}_p)$ as in Section 20.2, so quickly give its Def. 20.2.1 and Prop. 20.2.2 and Prop. 20.2.3.

Finish by constructing and explaining the étaleness of the Grothendieck–Messing period map π_{GM} for local shtuka spaces, using the aforementioned description of the Beilinson–Drinfeld Grassmannian (Prop. 23.3.3).

(IV) Local Shimura varieties as generic fibers of Rapoport–Zink spaces. [SW20]
In this talk, the proof of Theorem 24.2.5 should be explained and this Theorem should be used to deduce Theorem 1 from the introduction.

First, give the definition of local Shimura data as in Def. 24.1.1, also motivating the admissibility condition (cf. Prop. 24.1.2) and explaining the restriction to minuscule μ: Firstly, mention that all global Shimura data give (by definition) minuscule μ. Secondly, give Def. 19.4.1 and Prop. 19.4.2 on identifying the locally closed Schubert stratum $\mathrm{Gr}_{G,\mu}$ of the Beilinson–Drinfeld Grassmannian in the minuscule case with the diamond of the flag variety $\mathcal{F}\ell_{G,\mu}$ for the dynamic parabolic P_μ. You can black box this fact and do not have to give the construction of the Bialynicki–Birula map.

As a consequence of this fact, explain that one can upgrade the Grothendieck–Messing period map as introduced in the last talk, a priori an étale map of diamonds from a local shtuka space to the locally closed stratum $\mathrm{Gr}_{G,\mu}$ inside the Beilinson–Drinfeld Grassmannian, to an étale map of rigid spaces $\mathcal{M}_{G,b,\mu,K} \to \mathcal{F}\ell_{G,\mu}$ over $\breve{E}$ (using $X_{\text{ét}} \cong X^{\diamondsuit}_{\text{ét}}$).

Finally, carefully explain the proof of Theorem 24.2.5 that this local Shimura variety can be identified with the rigid generic fiber of a Rapoport–Zink space, and take time to mention statements of p-adic Hodge theory (like étaleness of the Grothendieck–Messing period map for Rapoport–Zink spaces, characterization of the fibers and the image) going into the proof. In particular, Section 5 of [SW13] should also be a helpful reference here. Deduce Theorem 1 from the more general version of Theorem 24.2.5 as in Corollary 24.3.5 (and using the duality of local shtuka data at infinite level from talk III) and explain why this duality interchanges the Hodge–Tate and Grothendieck–Messing period maps, cf. [SW13, Prop. 7.2.2]. For this, you will in particular have to explain what the Hodge–Tate period map constructed in Talk 1 for infinite-level Rapoport–Zink spaces corresponds to after taking the generic fiber, under the isomorphism of this generic fiber with the infinite-level local shtuka space.

(V) Application to the Jacquet–Langlands correspondence: A survey of [HM26].
Recall the classical local Jacquet–Langlands correspondence (as e.g. in [HM26, Theorem 1.1]) and note that its construction is somewhat inexplicit, relying on global trace formula methods. Then, briefly recall Scholze's construction of a Jacquet–Langlands functor $\mathcal{J}^i$ in [Sch18] (cf. also the introduction of [HM26]); in particular, state Theorem 1.1 (where you may black-box the precise definition of $\mathcal{F}_\pi$). Emphasize that its proof relies on his primitive comparison theorem and the properness of $\mathbb{P}^{n-1}_{\mathbb{C}_p}$, but that the latter is used rather indirectly through the elaborate usage of spectral sequences.

Then, turn to [HM26] and first recall the rough idea of the paper as explained in the introduction above. Explain the identification of $\mathrm{D}_{\text{ét}}([*/\underline{G}], \mathbb{F}_p) \cong \mathrm{D}(\mathrm{Rep}^{\mathrm{sm}}_{\mathbb{F}_p}(G))$ (cf. [HM24, Prop. 5.1.12] [HM26, Prop. 3.9]) and compare the functor $\mathcal{J}$ defined above with Scholze's Jacquet–Langlands functors $\mathcal{J}^i$. Explain (cf. [Man22, Section 1.2]) why this cohomology theory does not give a well-behaved six-functor-formalism, and then give the definition and key properties of the (stacky) six-functor-formalism of solid almost $\mathcal{O}^+/p$-sheaves (with φ-module structure), cf. [Man22, Theorem 2.2, Prop. 2.6, Prop. 2.9, Def. 3.9.10] and [HM26, Prop. 2.6, Prop. 2.9, Prop. 2.11]; in particular, explain that the map $f : * \to [*/G]$ is *p-fine* (and thus !-able) in the cases of our interest (cf. [HM26, Lemma 3.11, Corollary 3.12]), and satisfies $f_* = f_!$ for profinite G (of finite p-cohomological dimension). Mention also the criterion for smoothness of [HM26, Theorem 3.16] and the fact that it is satisfied for p-adic Lie groups (cf. [HM26, Theorem 3.18]). All the relevant notions are also discussed in [HM24]; we refer e.g.

to Prop. 5.3.2, Def. 5.3.12, Def. 5.3.17, Example 5.3.22. State the p-torsion Riemann–Hilbert correspondence of [Man22, Theorem 3.9.23].

Now, following the formalism of [HM24], identify f-suave objects in $\mathrm{D}([*/G], \mathbb{F}_p) = \mathrm{D}(\mathrm{Rep}^{\mathrm{sm}}_{\mathbb{F}_p}(G))$ with admissible representations. Give Lemma 4.5.16 of [HM24] and explain why we cannot explain it in the situation we desire. Finally, pass to the six-functor-formalism $\mathrm{D}^a_\square(\mathcal{O}^+_X/p)^\varphi$ and give the Definition 3.24 of [HM26] of admissibility in $\mathrm{D}^a_\square(\mathcal{O}^+_X/p)^\varphi$. Explain the characterization of Prop. 3.26 of [HM26] of admissible objects; you may also connect with f-suave objects for this six-functor-formalism. Explain the commutativity of the diagram connecting $\mathcal{J}$ and $\mathcal{J}_\square$ in the introduction and conclude (e.g. as in [HM26, Theorem 4.2(i)], or just using the criterion of [HM24, Lemma 4.5.16]).

Since there is a lot of content to be covered in this talk, the style of the talk should resemble a survey and it is more important to explain the ideas than to be extremely precise about all the statements. In particular, it is not necessary to formally introduce e.g. almost mathematics or abstract six-functor-formalisms.

Prerequisites

As the Kleine AG aims to prove a rather deep result in p-adic geometry, some knowledge of the basics of this field are assumed. In particular, the following might be helpful:

- Basic knowledge on adic and perfectoid spaces, e.g. as in Lectures 2–5, Lectures 6–7 of [SW20].
- Knowledge of the definition of diamonds and (locally) spatial diamonds; in particular, the example of $\mathrm{Spd}(\mathbb{Q}_p)$ as parametrizing untilts to characteristic 0 and the non-example of $\mathrm{Spd}(\mathbb{Z}_p)$ as parametrizing all untilts; the étale site of a diamond, diamondification of an adic space and the fact that $X_{\text{ét}} \cong X^\diamondsuit_{\text{ét}}$ (cf. Sections 8–10 of [SW20]).
- Basic understanding of p-divisible groups (which will only very briefly recalled in Talk 1). Also, a general familiarity with moduli problems and (1-)category theory is assumed.
- Basic knowledge in p-adic Hodge theory, e.g. having encountered the objects $A_{\mathrm{inf}}, B^+_{\mathrm{dR}}$ or the Fargues–Fontaine curve before might be beneficial. There are a lot of good lecture notes on this topic; we refer to the CMI summer school notes of Brinon–Conrad or the notes for the Math 679 course at U-Michigan on p-adic Hodge theory taught by Serin Hong.
- For the last talk, a broad idea of what a six-functor-formalism and !-ability should be and how the geometric properties of smoothness and properness are related to the identities $(-)^! = (-)^*$ and $(-)_! = (-)_*$ is helpful. We refer to the Introduction of [HM24].

References

[HM24] Claudius Heyer and Lucas Mann. *6-Functor Formalisms and Smooth Representations.* Preprint, arXiv:2410.13038 [math.CT] (2024). 2024. URL: https://arxiv.org/abs/2410.13038.

[HM26] David Hansen and Lucas Mann. "p-adic sheaves on classifying stacks, and the p-adic Jacquet-Langlands correspondence". English. In: *J. Inst. Math. Jussieu* 25.3 (2026), pp. 1441–1486. ISSN: 1474-7480. DOI: 10.1017/S1474748025101503.

[Man22] Lucas Mann. *A p-Adic 6-Functor Formalism in Rigid-Analytic Geometry.* Preprint, arXiv:2206.02022 [math.AG] (2022). 2022. URL: https://arxiv.org/abs/2206.02022.

[RZ17] Michael Rapoport and Thomas Zink. "On the Drinfeld moduli problem of p-divisible groups". English. In: *Camb. J. Math.* 5.2 (2017), pp. 229–279. ISSN: 2168-0930. DOI: 10.4310/CJM.2017.v5.n2.a2.

[RZ96] M. Rapoport and Th. Zink. *Period spaces for p-divisible groups.* English. Vol. 141. Ann. Math. Stud. Princeton, NJ: Princeton Univ. Press, 1996. ISBN: 0-691-02781-1; 0-691-02782-X. DOI: 10.1515/9781400882601.

[Sch18] Peter Scholze. "On the p-adic cohomology of the Lubin-Tate tower". English. In: *Ann. Sci. Éc. Norm. Supér. (4)* 51.4 (2018), pp. 811–863. ISSN: 0012-9593. URL: smf4.emath.fr/en/Publications/AnnalesENS/4_51/html/ens_ann-sc_51_811-863.php.

[SW13] Peter Scholze and Jared Weinstein. "Moduli of p-divisible groups". English. In: *Camb. J. Math.* 1.2 (2013), pp. 145–237. ISSN: 2168-0930. DOI: 10.4310/CJM.2013.v1.n2.a1.

[SW20] Peter Scholze and Jared Weinstein. *Berkeley lectures on p-adic geometry.* English. Vol. 207. Ann. Math. Stud. Princeton, NJ: Princeton University Press, 2020. ISBN: 978-0-691-20209-9; 978-0-691-20208-2; 978-0-691-20215-0. DOI: 10.1515/9780691202150.